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(Residential Autonomous College under University of Calcutta)

B.A./B.Sc. FIRST SEMESTER EXAMINATION, JANUARY 2015

FIRST YEAR

Date : 05/01/2015 **PHYSICS (Honours)**

Time : 11 am – 3 pm **Paper : I** Full Marks : 100

[Use a separate Answer Book for each group]

Group – A

(*Answer any seven questions taking atleast three from each Unit*) [7×10]

Unit – I

1. a) Show that for a given set of linearly independent vectors $\{\vec{a}_1, \vec{a}_2, \vec{a}_3\}$, an orthonormal basis $\{\vec{u}_1, \vec{u}_2, \vec{u}_3\}$ can be constructed such that $\vec{u}_1$ is a scalar multiple of $\vec{a}_1$, u_2 is linear combination of $\vec{a}_1$ and $\vec{a}_2$, and $\vec{u}_3$ is a linear combination of $\vec{a}_1, \vec{a}_2, \vec{a}_3$. [6]

   b) Given $\vec{a}_1 = (-1,1,1)$, $\vec{a}_2 = (1,-1,1)$, $\vec{a}_3 = (1,1,-1)$, construct the orthonormal basis $\{\vec{u}_1, \vec{u}_2, \vec{u}_3\}$, using 1(a). Hence, express the vector $\vec{A} = (1,2,3)$ in this basis. [2+2]

2. a) Find a power series solution about x = 0 of the differential equation

$$y'' - 2xy' + \lambda y = 0,\ \lambda \geq 0$$

   For what value of λ does this yield a polynomial solution? Hence find the solutions for $\lambda = 4$ and $\lambda = 6$. [6]

   b) If $\vec{r}$ is the position vector and $r = |\vec{r}|$, calculate the following :

   (i) $\vec{\nabla}.[f(r)\vec{r}]$ and (ii) $\vec{\nabla}.\vec{\nabla}[f(r)]$.

   If the first expression (i) is zero, find f(r). [4]

3. a) Given the circle C represented by $x^2 + y^2 = a^2,\ z = 0$, evaluate

   (i) $\oint_c \vec{r}.d\vec{r}$, (ii) $\oint_c \vec{r} \times d\vec{r}$,

   where $\vec{r}$ is the positive vector of any point on the circle. [3]

   b) Use Green's theorem for a plane to show that the area A of a region R in the x – y plane bounded by a simple closed curve c is,

$$A = \oint_c x\,dy = -\oint_c y\,dx = \frac{1}{2}\oint_c (x\,dy - y\,dx).$$ [3]

   c) i) Show that volume enclosed by any closed surface S is,

$$V = \frac{1}{6}\oiint_S \nabla(r^2).d\vec{s}.$$

   ii) Let S be a finite surface bounded by a simple closed curve c and ϕ a scalar field with continuous derivatives. Use Stokes' theorem to prove that

$$\iint_S d\vec{s} \times \nabla\phi = \oint_C \phi\, d\vec{r}.$$ [4]

4. a) If $\vec{f}$ and $\vec{g}$ are two vector functions, show that $(\vec{f} \times \vec{\nabla}).\vec{g} = \vec{f}.(\vec{\nabla} \times \vec{g})$. [3]

   b) Find the eigenvalues and eigenvectors of the matrix

$$A = \begin{pmatrix} -4 & -6 \\ 3 & 5 \end{pmatrix}$$

   Hence diagonalise A. [4]

   c) Find the directional derivative of $\phi(x,y,z) = x^2 + y^2 + z^2$ in the direction from P(1,1,0) to Q (2,1,1). Also find its maximum value and direction at (1,1,0). [3]

5. a) Set up the differential equation of a forced harmonic oscillator in the presence of a damping force proportional to instantaneous speed and an external periodic force $F = F_0 \cos\omega t$. Obtain the steady-state solution.
Hence, find the frequency of amplitude resonance. [6]

b) Show that for a forced vibration, the total energy of the vibrating system is not constant. Prove that for such a case,

$$\frac{\text{Average potential energy}}{\text{Average kinetic energy}} = \frac{\omega_0^2}{\omega^2},$$

where ω_0 is the natural frequency of the undamped oscillator. [4]

Unit – II

6. a) Obtain expressions for the velocity and acceleration of a particle in cylindrical polar coordinates (ρ, ϕ, z). Hence show that if the particle moves in the x – y plane under the action of a radical force, its angular momentum is constant. [6]

b) A particle moves along a space curve $\vec{r} = \vec{r}(s)$, where s is the arc-length parameter. Show that the acceleration vector $\vec{a}(t)$ is given by,

$$\vec{a}(t) = \frac{dv}{dt}\hat{T} + \frac{v^2}{\rho}\hat{N},$$

where v is the speed at any instant, and $\hat{T}$ and $\hat{N}$ are the unit tangent and unit normal to the curve at that instant, and ρ is the radius of curvature. [4]

7. a) Let S' be an inertial frame moving with uniform velocity $\vec{v}$ relative to an inertial frame, S. If, in a two-particle collision, the total linear momentum is, conserved in S. Show that it is also conserved in S'. [3]

b) State and prove the work-energy theorem for a particle moving in a force field, $\vec{F}$. If $\vec{F}$ is a conservative force, show that a potential energy function V can be defined such that the total mechanical energy E = T+V is conserved, when T is the kinetic energy. [4]

c) Suppose the position vector of a particle is given by, $\vec{r} = \hat{i}\, b\sin\omega t + \hat{j}\, b\cos\omega t + \hat{k}c$.
Show that,
i) the velocity vector is parallel to the x–y plane;
ii) distance from the origin remains constant;
iii) the acceleration is perpendicular to the velocity. [3]

8. a) A rocket starts falling vertically from rest under constant gravity. From the instant it starts falling. From the instant it starts falling, it is also ejecting gas at a constant rate α in the vertically downward direction at a speed v_0 with respect to the rocket. Determine the speed of a the rocket after time t. How far the rocket will travel in this time? [6]

b) Show for a rocket staring initially from rest with $v_{rel} = 2072\cdot 64\,ms^{-1}$ (velocity of escaping gas with respect to the rocket vertically downward) and a mass loss per second equal to $\frac{1}{60}$th of the initial mass M_0 that in order to reach the escape velocity $(v_e = 11\cdot 2Km\,sec^{-1})$ from the earth the ratio of the mass of the fuel $(M_0 - M')$ to the mass of the empty rocket (M') must be almost 300. [4]

9. a) A particle is projected vertically upwards with initial speed u in a medium which exerts a resistance kv^2 per unit mass, where v is the instantaneous speed and k is a constant. Prove that the maximum height reached is given by

$$H = \frac{1}{2k}\ln\left(1 + \frac{ku^2}{g}\right).$$

Also find the time T when the particle reaches the highest point. [6]

b) A raindrop falls from rest at a place where the air resistance is proportional to the velocity v. Find the velocity, acceleration and distance travelled by the raindrop in time t. What is its limiting velocity? [4]

10. a) A system of N mass points $m_i (i = 1, 2, ..., N)$ moves under the action of external forces $\vec{F}_i$ and pairwise internal forces $\vec{f}_{ij}$. The latter obeys Newton's third law in both weak and strong form.

i) Find the velocity of the centre of mass (cm), and hence show that the cm moves with a constant velocity if the total external force is zero.

ii) Show that the time rate of change of the angular momentum of the system is equal to the total external torque.

iii) Show the Kinetic energy of the system is given by the same of the kinetic energy of translation of the cm plus the kinetic energy of the particles relative to the cm. [6]

b) If two bodies undergo a direct (head-on) collision, show that the loss in kinetic energy is equal to

$\Delta T = \frac{1}{2}\mu v^2 (1 - \epsilon^2)$,

where μ = reduced mass, v = relative speed before impact, ϵ = coefficient of restitution. [4]

Group – B

(*Answer any three questions*) [3×10]

11. a) Using Fermat's principle, establish the laws of reflection on a concave surface. [4]

b) Deduce the one dimensional ray equation for a light ray in an inhomogenous medium where the refractive index is a function of x only.

From this equation show that the ray path in a homogenous medium is a straight line. [3+1]

c) Show that all the rays passing through one focus of an elliptic surface pass through the other focus after reflection. [2]

12. a) Define principal and focal points of an optical system. Show that when a thick lens is surrounded by a homogeneous medium the nodal points coincide with the principal points. [2+2]

b) Determine the basic refraction matrix through a spherical surface. [3]

c) Consider a sphere of radius 20 cm and the refractive index of the material is 1·6. Obtain the positions of its focal points. The sphere is placed in air. [3]

13. a) What is aplanatic foci of an optical system? Find the position of aplanatic foci of a spherical refracting surface. State one of its application. [1+3+1]

b) How can spherical aberration be removed using two convex lenses separated by a distance? [3]

c) A convergent doublet of separated lenses, corrected for a spherical aberration, has an equivalent focal length of 10 cm. The lenses are separated by 2 cm. What are the focal lengths of the component lenses? [2]

14. a) Define dispersive power of a transparent medium. What are axial and lateral chromatic error of a lens? [1+2]

b) What do you mean by telescopic system? Express lateral magnification of a telescopic system in terms of its angular magnification. [1+2]

c) An achromatic telescope objective of 1·5 m focal length consists of two thin lenses in contact. Their dispersive powers are 0·05 and 0·075 respectively. Calculate their focal lengths. Is it possible to construct the objective using same type of glass lens? [3+1]

15. a) What is an eyepiece? Why should it consists of two lenses? [1+1]

b) Draw the path rays within a Ramsden's eye-piece and indicate the position of cross-wire. [2]

c) Find the focal points and principal points of a Ramsden's eyepiece. [4]

d) What do you mean by entrance and exit pupil? [2]

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